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a) The local train departures every 5 minutes. We arrive randomly and independently of the trains, and consequently we are equally likely to arrive at any point in interval 0 to 5 minutes before a departure.

Let W_L and W_E denote the waiting time from arrival at the platform to the departure of the first local and express train, respectively. Then

$W_E \sim U(0,15)$ (w. 0 included),
$W_L \sim U(0,5)$ (w. 0 included).

b) The next local train arrives alone (i.e. not simultaneously with an express train) 2 out of 3 times. Hence, the probability is 2/3.

If the next two local trains should arrive alone, it requires that the local (last) train that depatured before our arrival depatured simultaneously with an express train. This has happened with probability 1/3, which is the answer to the question.

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c) Given the next train arrives alone there is a 50% chance that the subsequent train (local) arives alone too. We get this from the rule $\mathbb{P}(A|B) = \mathbb{P}(AB)/\mathbb{P}(B)$, which in our case simplifies to $(1/3)/(2/3) = 1/2$.

Our travel times can take two forms

- We board the local train: 17 minutes
- We wait for the next train: (5 min.)
  - If it arrives alone, we wait for the next train, which will not arrive alone: 5 + 11 minutes.
  - If it does not arrive alone, we board the express train: 11 minutes

Thus, the expected travel time for option one (boarding) is 17 minutes. For the second option (waiting) the expected travel time is:

$$\frac{1}{2}\cdot(5+11) + \frac{1}{2}\cdot(5+[5+11]) = 18.5$$

In conclusion, if the local train arrives alone, we should board it.

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d) Now define T as the total travel time from our arrival at the platform. We invoke the strategy from subproblem c. First, we introduce the indicator variable I_A, which signals whether the next local train arrives alone. Then,

$$T = W_L + I_A \cdot 17 + (1 - I_A) \cdot 11.$$

Hence,

$$\mathbb{E}[T] = \mathbb{E}[W_L] + \mathbb{E}[I_A]17 + \mathbb{E}[1 - I_A]11$$
$$= 5/2 + 1/3 \cdot 11 + 2/3 \cdot 17 =$$
$$= 105/6 = 17.5.$$

e) We have two options:

1. Wait at the platform for local trains: $T_L = W_L + 17$,

2. Wait at the platform for express trains: $T_E = W_E + 11$,

$$\mathbb{E}[T_L] = \mathbb{E}[W_L] + 17 = 5/2 + 17 = 19.5,$$
$$\mathbb{E}[T_E] = \mathbb{E}[W_E] + 11 = 15/2 + 11 = 18.5.$$

Thus, we choose the express platform.