Exercise 7.4.1

We shall apply eq. (7.17). In order to do so, we must calculate μ and σ for life time distribution.

$$\mu = \int_0^1 x \cdot 2x \, dx = \left[\tfrac{2}{3}x^3\right]_0^1 = \tfrac{2}{3},$$

$$\begin{aligned}\sigma^2 &= \int_0^1 x^2 \cdot 2x \, dx - \mu^2 \\ &= \left[\tfrac{1}{2}x^4\right]_0^1 - \left(\tfrac{2}{3}\right)^2 \\ &= \tfrac{1}{2} - \tfrac{4}{9} = \tfrac{1}{18}.\end{aligned}$$

Thus, eq. (7.17) yields that

$$M(t) - \frac{t}{\mu} \approx \frac{\sigma^2 - \mu^2}{2\mu^2},$$

which is

$$M(t) - \tfrac{3}{2}t \approx -\tfrac{7}{16}.$$

Thus,

$$M(t) \approx \tfrac{3}{2}t - \tfrac{7}{16}$$

asymptotically.