

Exercise 7.2.1

In this case, a renewal interval consists of two independent geometrically distributed random variables, specifically:

- The time spent in state 0, say $N_0 \sim \text{geo}(a)$
- The time spent in state 1, say $N_1 \sim \text{geo}(b)$

Thus, if X_1 is the first renewal time, we have

$$X_1 = N_0 + N_1.$$

Hence,

$$\mathbb{E}[X] = \mathbb{E}[N_0 + N_1] = \mathbb{E}[N_0] + \mathbb{E}[N_1] = 1/a + 1/b,$$

due to the linearity of the expectation operator. Due to the independence of N_0 and N_1 , we find the variance as

$$\mathbb{V}[X_1] = \mathbb{V}[N_0 + N_1] = \mathbb{V}[N_0] + \mathbb{V}[N_1] = (1-a)/a^2 + (1-b)/b^2.$$

You can consult sec. 1.3.3 for the results on the geometric distribution.