

Problem 7.1.3

The residual lifetime is defined on p. 349 as

$$Y_t = W_{N(t)+1} - t.$$

Hence,

$$\mathbb{E}[Y_t] = \mathbb{E}[W_{N(t)+1} - t] = \mathbb{E}[W_{N(t)+1}] - \mathbb{E}[t],$$

due to the linearity of the expectation operator. The left term is given in the problem, while $\mathbb{E}[t] = t$. So,

$$\mathbb{E}[Y_t] = \mathbb{E}[X_1](1 + M(t)) - t. \quad \square$$