

Wolff 2.33

We consider a sample of 1,000 women and assume that the sample size is sufficiently large that the population probabilities (α_j , $j=1,2,3$) applies approximately.

- a) Consider a population of N women. According to the offspring distribution $\{\alpha_j\}$, a third of the women will have 1 child, a third will have 2 children, and a third will have 3 children. This means that theoretically, the next generation should consist of $N/3 \cdot 1 + N/3 \cdot 2 + N/3 \cdot 3 = 2N$ individuals / children, of which:
- $N/3 \cdot 3 = N$ should have a mother w. 3 children,
 - $N/3 \cdot 2 = \frac{2}{3}N$ should have a mother w. 2 children,
 - $N/3 \cdot 1 = \frac{1}{3}N$ should have a mother w. 1 children.

In other words, $N/2N = 1/2$ of the offspring will have a mother w. 3 children, $\frac{2}{3}N/2N = 1/3$ of the offspring will have a mother w. 2 children and so on. Thus, the average response to Q1 is $1/2 \cdot 3 + 1/3 \cdot 2 + 1/6 \cdot 1 = 14/6 = 7/3$.

- b) This is considerably easier as this is simply the mean of the offspring distribution $\{\alpha_j\}$, which is .2.

- c) The difference simply stems from the fact that we average on two different bases (i.e. with different weights). In a) we take an average based on offspring, while in b) we take an average based on parents. As we sample among offsprings in a), and sample among parents in b), the weights in the calculation of the averages will change. For instance, a mother with 3 children has the same probability of being interviewed as a mother with 1 child in b), but this is not the case in a). In a) every mother with 3 children has triple the probability of having a child interviewed as ~~as~~ a mother with 1 child. Therefore, increasing the sample size would not change this, but the results would move towards the theoretical values. Instead, if the averages should coincide, we should change the weights in a) by ensuring that none of the interviewees have the same mother. In that way, a mother w. 3 children and a mother with 1 child would have the same probability of having a child interviewed.