Problem 9.2.2

We consider two queueing systems.

System 1: $M/M/2$, $\lambda = 2$, $\mu = 1.2$,

System 2: $M/M/1$, $\lambda = 1$, $\mu = 1.2$.

For system 1: The traffic intensity $\rho = \lambda / s\mu$ must be less than one for the system to be stable. In this case, we get

$$\pi_0 = \left(\sum_{j=0}^{s-1} \frac{1}{j!} \left(\frac{\lambda}{\mu}\right)^j + \frac{(\lambda/\mu)^s}{s!(1-\lambda/s\mu)} \right)^{-1} \quad \text{(Top of p. 458)}$$

$$= \left(1 + \left(\frac{2}{1.2}\right)^1 + \frac{(2/1.2)^2}{2!(1-2/2\cdot 1.2)} \right)^{-1}$$

$$= 1/11.$$

We can then calculate the mean number of customers in the queue (not the entire system!) using eq. (9.19)

$$L_Q = \frac{\pi_0}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{(\lambda/s\mu)}{(1-\lambda/s\mu)^2} = \frac{1}{11} \cdot \frac{1}{2!} \left(\frac{2}{1.2}\right)^2 \frac{(2/2\cdot 1.2)}{(1-2/2\cdot 1.2)^2}$$

$$= 125/33.$$

Hence, $W_Q = L_Q/\lambda = 125/66$, and thus

$$W = W_Q + \frac{1}{\mu} = 125/66 + 5/6 = 30/11.$$

Problem 9.2.2

For system 2: If the traffic intensity $\rho = \lambda / \mu$ is less than one, the system is stable and we can use eq. (9.15) to find

$$W = (\mu - \lambda)^{-1} = (6/5 - 1)^{-1} = (1/5)^{-1} = 5.$$

The reason for this result is related to the variability in the two systems. In system 2, there is only one server and consequently a long service is more likely to effect a large number of customers. Essentially, system 1 is less likely to be "blocked" by long service times as the customers can be served by the other server.