Problem 6.2.3

We consider a pure death process $\{X_t\}_{t\geq 0}$ with parameters μ_i for $i = 1, \ldots, N$ and $X_0 = N$. Note that state 0 is absorping.

Consider the process as follows: See Figure 6.1 or Figure 6.2. For a duration of S_N the process is a state N, which implies that the area under the trajectory is NS_N. Similar arguments apply for the other states and consequently

$$\int_0^\infty X_t \, dt = \sum_{i=1}^{N} i S_i$$

Hence,

$$\mathbb{E}\left[\int_0^\infty X_t dt\right] = \mathbb{E}\left[\sum_{i=1}^N i S_i\right] = \sum_{i=1}^N i \, \mathbb{E}[S_i]$$

$$= \sum_{i=1}^N i \mu_i^{-1}.$$

Note that you get the same result by using the hint from the book. The hint simply states that $\sum_{i=1}^N W_i = \sum_{i=1}^N i S_i$, which basically is the result from ~~problem~~ exercise 6.1.2 b).