Exercise 6.3.1

For $\{X_t\}_{t\geq 0}$ arrivals are dictated by a Poisson arrival process and consequently we know that the arrival rate is constant. We also know that we can only increase by one per increment.

Each particle has an exponentially distributed lifetime independent of the remaining (other) particles. Therefore, we only decrease the number of particles by more than one if two or more particles decay at exactly the same time, which only happens with probability 0.

Finally, note that $X_t \geq 0$ and therefore $\mu_0 = 0$. In conclusion, all 5 postulates on p. 295 are satisfied. Hence, $\{X_t\}_{t\geq 0}$ is a birth and death process. Due to the Poisson arrival process, we have $\lambda_n = \lambda$, i.e. the arrival rate is constant. In state n, there are n particles, which all decay at rate μ. Hence, the total rate is $\mu_n = n\mu$ for $n > 0$ and $\mu_0 = 0$.