Exercise 5.3.2

Let $\{X_t\}$ denote the number of emitted particles until time t. Then $\{X_t\}_{t\geq 0}$ is a Poisson process with rate $\lambda = 2$ particles per minute.

a) Recall that increments in non-overlapping intervals are independent. Thus,

$$\mathbb{P}(X_5 - X_3 > 0, X_3 - X_0 = 0)$$

$$= \mathbb{P}(X_5 - X_3 > 0)\,\mathbb{P}(X_3 - X_0 = 0)$$

$$= \mathbb{P}(X_3 - X_0 = 0)\,(1 - \mathbb{P}(X_5 - X_3 = 0))$$

$$= e^{-6}(1 - e^{-4}) = e^{-6} - e^{-10}.$$

b) I'm not sure what the problem is asking (whether to include $X_3 - X_0 = 0$ or not), so I'll provide two answers.

1) $\mathbb{P}(X_5 - X_3 = 1) = (2\cdot 2)^1 e^{-2\cdot 2}/1! = 4e^{-4}$.

2) $\mathbb{P}(X_5 - X_3 = 1, X_3 - X_0 = 0)$

$$= \mathbb{P}(X_5 - X_3 = 1)\,\mathbb{P}(X_3 - X_0 = 0)$$
$$= \left((2\cdot 2)^1 \cdot e^{-2\cdot 2}/1!\right)\left((2\cdot 3)^0 \cdot e^{-2\cdot 3}/0!\right)$$
$$= 4e^{-4}e^{-6} = 4e^{-10}.$$