

Exercise 5.1.1

Let $\{X_t\}_{t \geq 0}$ denote the number of defects along the filament. Then $\{X_t\}$ is a CTMC, Specifically a Poisson process with rate $\lambda = 2$ defects per foot.

a) Note that X_t is the number of defects in the interval $[0, t]$. Thus, we seek the probability $P(X_1 = 0)$. Using that $X_0 = 0$ and the definition on p. 225 - 226, we get:

$$\begin{aligned} P(X_1 - X_0 = 0) &= P(X_1 = 0) = (2 \cdot 1)^0 \cdot e^{-2 \cdot 1} / 0! \\ &= e^{-2}. \end{aligned}$$

b) To calculate the probability of interest, recall that non-overlapping intervals have independent increments. Moreover, increments over non-overlapping intervals of equal length have identical distributions. Hence,

$$\begin{aligned} P(X_2 - X_1 = 0 \mid X_1 - X_0 = 1) &= P(X_2 - X_1 = 0) \\ &= P(X_1 - X_0 = 0) \\ &= e^{-2}. \end{aligned}$$