Problem 5.3.7

Let $\{X_t\}_{t\geq 0}$ denote the number of failed components up till time t. Then $\{X_t\}_{t\geq 0}$ is a Poisson process with rate $\lambda = 2$ per year.

We seek the minimum number of (k) components such that

$\mathbb{P}(X_1 \geq k) < 0.02.$

This is equivalent to finding the minimum k such that

$\mathbb{P}(W_k \leq 1) < 0.02.$

Here $W_k \sim \gamma(k, 2)$ (Gamma$(k,2)$). Thus:

$$\mathbb{P}(W_k \leq 1) = 1 - \sum_{i=0}^{k-1} \frac{(2 \cdot 1)^i e^{-2 \cdot 1}}{i!}.$$

The results are as follows

k	1	2	3	4	5	6	7
$\mathbb{P}(W_k \leq 1)$	0.86	0.59	0.32	0.14	0.05	0.02	0.00

For $k=6$;
$\mathbb{P}(W_k \leq 1) = 0.017.$

In conclusion, the submarine should bring at least six components, i.e. 5 spares + 1 original, to comply with the criteria.