Exercise 4.1.1

We consider a DTMC $\{X_n\}_{n \in \mathbb{N}_0}$ governed by the transition probability matrix

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}.$$

In order to apply Theorem 4.1, we need to check if P is regular. We check the two conditions on p. 168.

1. All states communicate.
2. At least one state has period 1.

As P satisfies the conditions, we can use Theorem 4.1. Hence:

$$\begin{aligned} \pi_0 &= 0.7\pi_0 + 0 \cdot \pi_1 + 0.5\pi_2, \\ \pi_1 &= 0.2\pi_0 + 0.6\pi_1 + 0 \cdot \pi_2, \\ \pi_2 &= 0.1\pi_0 + 0.4\pi_1 + 0.5\pi_2, \\ \pi_0 + \pi_1 + \pi_2 &= 1. \end{aligned}$$

The solution to the system is $(\pi_0, \pi_1, \pi_2) = (10/21, 5/21, 6/21)$.