Exercise 4.3.1

To identify the equivalence classes, we examine communication between the states.

Note that we can have the path $0 \to 1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 0$. (with probability greater than zero). Since, the process circles through all states, we can conclude that all states communicate. Hence communication induces only one equivalence class.

We note that the process can circle from state zero and back in either 5 or 8 steps. Therefore, $P_{00}^{(n)}$ is greater than zero for all
$(n \geq 1)$ n which can be expressed as a sum of non-negative multiples of 5 and 8. Hence, $P_{00}^{(n)} > 0$ for $n = 5, 8, 10, 13, 15, 16, 18, 20$.

Since there is only one class, it follows from p. 197 that all states have the same period. Using the definition in the beginning of sec. 4.3.2 and the result above, we get

$$d(0) = \gcd\{n \geq 1 : P_{00}^{(n)} > 0\} = \gcd\{5, 8, \ldots\} = 1.$$

Hence, all states are aperiodic. In conclusion, the Markov chain is aperiodic.