# Problem 3.8.3

Let $F$ be a random variable indicating the sex of the first born child. Furthermore, let $X$ denote the total number of children the family gets. Let $p$ be the probability that a child is a girl.

$$X = I\{F = G\} \cdot 2 + I\{F = B\} \cdot (X_B + 1),$$

where $X_B$ denotes the number of boys following the first born. Thus, $X_B \sim Geo(p)$ (on $\{1, \dots\}$)

$\mathbb{P}(X=0) = 0$, $\mathbb{P}(X=1) = 0$,

$$\mathbb{P}(X=2) = \mathbb{P}(X=2 \mid F=G)\mathbb{P}(F=G) + \mathbb{P}(X=2 \mid F=B)\mathbb{P}(F=B)$$
$$= 1 \cdot p + qp. = p(1+q).$$

$$\mathbb{P}(X=3) = \mathbb{P}(X=3 \mid F=G)\mathbb{P}(F=G) + \mathbb{P}(X=3 \mid F=B)\mathbb{P}(F=B)$$
$$= 0 \cdot p + \mathbb{P}(X_B=2)q = qpq = q^2p.$$

In general, for $k \geq 3$

$$\mathbb{P}(X=k) = q^{k-1}p.$$

# Problem 38.3

Let $N$ be the total number of boys. Then

$N = N_G I\{F=G\} + (N_B+1) I\{F=B\}$,

where $N_G \sim Ber(q)$ and $N_B \sim Geo(p)$.
(on $\{0, 1, \dots\}$)

Hence

$$\mathbb{P}(N=0) = \mathbb{P}(N=0 \mid F=G)\mathbb{P}(F=G) + \mathbb{P}(N=0 \mid F=B)\mathbb{P}(F=B)$$
$$= \mathbb{P}(N_G=0)p = p^2,$$

$$\mathbb{P}(N=1) = \mathbb{P}(N=1 \mid F=G)\mathbb{P}(F=G) + \mathbb{P}(N=1 \mid F=B)\mathbb{P}(F=B)$$
$$= \mathbb{P}(N_G=1)p + \mathbb{P}(N_B=0)q$$
$$= qp + pq = 2pq,$$

$$\mathbb{P}(N=2) = \mathbb{P}(N_B=1)q = qpq = q^2p,$$

$$\mathbb{P}(N=3) = q^3p,$$

$$\mathbb{P}(N=k) = q^kp.$$