# Problem 2.3.4

Let $\varepsilon_1, \varepsilon_2, \ldots$ be i.i.d. random variables with $\mathbb{E}[\varepsilon_i] = \mu$ and $\mathbb{V}[\varepsilon_i] = \sigma^2$. Then we form $S_N = \sum_{i=1}^{N} \varepsilon_i$.

a) Let $N \sim \text{Pois}(\lambda)$. Then $\mathbb{E}[N] = \lambda$ and $\mathbb{V}[N] = \lambda$. Applying eq. (2.30) yields

$$\mathbb{E}[S_N] = \mu\lambda, \quad \mathbb{V}[S_N] = \lambda\sigma^2 + \mu^2\lambda = \lambda(\sigma^2 + \mu^2).$$

b) Let $N \sim \text{Geo}(p)$. Then $\mathbb{E}[N] = \frac{1-p}{p} = \lambda$ and $\mathbb{V}[N] = \frac{1-p}{p^2} = \frac{\lambda}{p}$. Applying eq. (2.30):

$$\mathbb{E}[S_N] = \mu\lambda, \quad \mathbb{V}[S_N] = \lambda\sigma^2 + \mu^2\frac{\lambda}{p} = \lambda(\sigma^2 + \mu^2 p^{-1}).$$

Note that $\mathbb{V}[N] = \frac{1-p}{p} \cdot \frac{1}{p} = \lambda \cdot (\lambda + 1)$.

So $\mathbb{V}[S_N] = \lambda\sigma^2 + \mu^2\lambda(\lambda+1) = \lambda(\sigma^2 + \mu^2) + \mu^2\lambda^2$.

c) For $\lambda \to \infty$, we see that in

a) $\mathbb{V}[S_N] = O(\lambda)$

b) $\mathbb{V}[S_N] = O(\lambda^2)$.

In conclusion the variance growths much faster for the geometric sum than for the Poisson sum.