Exercise 3.7.1

We consider the model with

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/10 & 2/10 & 5/10 & 2/10 \\ 1/10 & 2/10 & 6/10 & 1/10 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

We first find

$$(I-Q)^{-1} = \begin{pmatrix} 20/11 & 25/11 \\ 10/11 & 40/11 \end{pmatrix}.$$

a) The above matrix is the fundamental matrix associated with Q, $W = (I-Q)^{-1}$.

We can use W to find U with the formula on the bottom of p. 143. $U = WR$.

In this case,

$$R = \begin{bmatrix} 1/10 & 2/10 \\ 1/10 & 1/10 \end{bmatrix}.$$

$$WR = \begin{bmatrix} 20/11 & 25/11 \\ 10/11 & 40/11 \end{bmatrix} \begin{bmatrix} 1/10 & 2/10 \\ 1/10 & 1/10 \end{bmatrix} = \begin{bmatrix} 9/22 & 13/22 \\ 5/11 & 6/11 \end{bmatrix}.$$

(For total time: $Y = W1$)