Danmarks Tekniske Universitet
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DTU

Problem 3.1.4

We have $\varepsilon_i \sim F$ i.i.d. such that

k	0	1	2	3
$\mathbb{P}(\varepsilon_i = k)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{4}{10}$.

Also, we have $X_0 = 0$ and $X_n = \max\{\varepsilon_1, \dots, \varepsilon_n\}$.

Then $\{X_n\}_{n\in\mathbb{N}_0}$ is a DTMC since ~~$X_n = \max\{X$~~

$X_n = \max(X_{n-1}, \varepsilon_n)$.

We get the transition probabilities as:

$\mathbb{P}(X_{n+1} = 0 \mid X_n = 0) = \mathbb{P}(\max(X_n, \varepsilon_{n+1}) = 0 \mid X_n = 0)$

$= \mathbb{P}(\max(0, \varepsilon_{n+1}) = 0) = \mathbb{P}(\varepsilon_{n+1} \leq 0) = \frac{1}{10}$.

We obtain:

$$P = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0.1 & 0.3 & 0.2 & 0.4 \\ 0 & 0.4 & 0.2 & 0.4 \\ 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$