Danmarks Tekniske Universitet
Technical University of Denmark

DTU

Problem 3.2.1

We shall prove the hypothesis using a proof by induction.

i) Induction hypothesis: $\mathbb{P}(X_n = k) = \frac{1}{4}$ for $k = 1, \dots, 4$.

ii) Base case: $n = 0$.

This follows directly from the assumption on the initial distribution.

iii) Induction step.

We assume that the hypothesis is true for some $n \in \mathbb{N}_0$.

$$\mathbb{P}(X_{n+1} = j) = \sum_{k=0}^{3} \mathbb{P}(X_{n+1} = j \mid X_n = k)\,\mathbb{P}(X_n = k)$$

$$= \frac{1}{4} \sum_{k=0}^{3} \mathbb{P}(X_{n+1} = j \mid X_n = k)$$

$$= \frac{1}{4}, \quad j = 0, \dots, 3.$$

iv) Conclusion

This is due to the doubly stochastic matrix and the uniform initial distribution.