Exercise 3.4.6

We follow the procedure from sec. 3.4.1.

a) We seek $u_1 = P(X_T = 0 \mid X_0 = 1)$, where

$T = \min \{n \geq 0 : X_n = 0 \vee X_n = 3\}$. To find u_1,

we need to solve the system:

$u_1 = 0.1 + 0.4 \cdot u_1 + 0.1 \cdot u_2$
$u_2 = 0.2 + 0.1 \cdot u_1 + 0.6 \cdot u_2$, $(u_2 = P(X_T = 0 \mid X_0 = 2))$

Hence, $0.4 u_2 = 0.2 + 0.1 u_1$ and consequently

$u_1 = 0.1 + 0.4 \cdot u_1 + 0.1 \cdot \left(\frac{0.2 + 0.1 \cdot u_1}{0.4} \right)$, which

simplifies to $0.6 u_1 = 0.1 + 0.05 + 0.025 u_1$,

and thus $u_1 \stackrel{\sim}{=} 0.261$ $(= 0.26087) = \frac{6}{23}$.

b) We use obvious notation:

$V_1 = 1 + 0.4 \cdot V_1 + 0.1 \cdot V_2$
$V_2 = 1 + 0.1 \cdot V_1 + 0.6 \cdot V_2$

$V_1 = \frac{50}{23}$, $V_2 = \frac{70}{23}$.

(!) Question is incomplete.