

Exercise 3.4.1

Let $T = \min \{n \geq 0 : X_n = 3\}$.

Then we seek $v_0 = \mathbb{E}[T | X_0 = 0]$.

Hence, we need to solve the system

$$v_0 = 1 + \frac{4}{10} \cdot v_0 + \frac{8}{10} \cdot v_1 + \frac{2}{10} \cdot v_2 \quad (1)$$

$$v_1 = 1 + 0 \cdot v_0 + \frac{7}{10} \cdot v_1 + \frac{2}{10} \cdot v_2 \quad (2)$$

$$v_2 = 1 + 0 \cdot v_0 + 0 \cdot v_1 + \frac{9}{10} \cdot v_2 \quad (3)$$

Thus, (3) yields

$$v_2 = 10 \quad (\text{you can think about a geo. dist.})$$

It follows that $v_1 = 10$ from (2) and that

$$v_0 = 10 \quad \text{from (1).}$$