

### Problem 3.4.7

$$h_i = \mathbb{E} \left[ \sum_{n=0}^{\infty} \beta^n c(x_n) \mid x_0 = i \right]$$

$$= \mathbb{E} \left[ c(x_0) + \sum_{n=1}^{\infty} \beta^n c(x_n) \mid x_0 = i \right]$$

$$= c(i) + \mathbb{E} \left[ \sum_{n=1}^{\infty} \beta^n c(x_n) \mid x_0 = i \right]$$

$$= c(i) + \sum_j \mathbb{E} \left[ \sum_{n=1}^{\infty} \beta^n c(x_n) \mid x_0 = i, x_1 = j \right] p_{ij}$$

$$= c(i) + \sum_j \mathbb{E} \left[ \sum_{n=1}^{\infty} \beta^n c(x_n) \mid x_1 = j \right] p_{ij}$$

$$= c(i) + \sum_j \mathbb{E} \left[ \sum_{n=0}^{\infty} \beta^{n+1} c(x_{n+1}) \mid x_1 = j \right] p_{ij}$$

$$= c(i) + \sum_j \beta \mathbb{E} \left[ \sum_{n=0}^{\infty} \beta^n c(x_n) \mid x_0 = j \right] p_{ij}$$

$$= c(i) + \beta \sum_j h_j p_{ij}$$