Problem 8.4.5

We consider a Brownian motion with negative drift $\mu < 0$, $\{X_t\}_{t \geq 0}$. We then construct a new process $\{X_t^A\}_{t \geq 0}$, which is defined as described in the book.

The probability that $\{X_t^A\}$ ever reaches the level b must be the same as the probability that it will reach level b prior to reaching level 0 as the process is then absorbed. Prior to absorption, $\{X_t^A\} = \{X_t\}$, and consequently if we define

$$\tau = \inf\{t \geq 0 : X_t = 0\}, \quad T = \inf\{t \geq 0 : X_t \in \{0, b\}\},$$

such that $T \leq \tau$, we have that $X_T^A = X_T$.

The probability that $\{X_t^A\}$ ever crosses the level b is now $\mathbb{P}(X_T^A = b)$. (we assume $X_0^A = x > 0$).
Theorem 8.1 (p. 420) yields

$$\mathbb{P}(X_T^A = b) = (e^{-2\mu x/\sigma^2} - 1)/(e^{-2\mu b/\sigma^2} - 1).$$