Problem 8.4.1

Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion and define $X_t = B_t - bt$. Since $b>0$, $\{X_t\}$ is a Brownian motion with negative drift.

For each $t \geq 0$, the events $\{B_t > a+bt\}$, and $\{B_t - bt > a\}$ are equivalent. As $X_t = B_t - bt$, we have that for each $t\geq 0$ $\{B_t > a+bt\} = \{X_t > a\}$.

In other words, the event that $\{B_t\}$ ever crosses the line $a+bt$ is equivalent with the event that $\{X_t\}$ ever crosses a (the level a).

Since $a>0$, eq. (8.48) applies and we get that

$$\mathbb{P}(\max_{t\geq 0} X_t > a) = e^{-2|-b|a/\sigma^2}.$$

In this case, the variance parameter $\sigma^2 = 1$ (it is NOT the variance itself). In conclusion,

$$\mathbb{P}(\max_{t\geq 0} X_t > a) = e^{-2ab}.$$