Problem 8.3.6

We consider an absorbed Brownian motion $\{A_t\}_{t\geq 0}$ with $A_0 = x$. Moreover, we define $M = \max_{t>0}\{A_t\}$.

The event $\{M \geq z\}$ $(0 < x < z)$ can only occur if A_t reaches the level z before it reaches 0, where it will be absorbed. Prior to absorption, A_t behaves exactly like a standard Brownian motion initiated at x, i.e. $\{B_t\}_{t\geq 0}$ with $B_0 = x$.

From the example on p. 397, we know that the probability that $\{B_t\}$ reaches z before 0 conditioned on $B_0 = x$ is exactly $1 - (z-x)/z$. We obtained this as we are looking for the complementary event compared to eq. (8.13). In conclusion,

$$\mathbb{P}(M \geq z) = 1 - (z-x)/z = x/z.$$