Course: Problem 8.2.1

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We will define $\nu(t^*, s)$ as in eq. (8.25), i.e. the probability that a standard Brownian motion $\{B_t\}_{t \geq 0}$ has a zero in the time interval $(t^*, t^*+s]$.

We then define two events: for $0 < a < b$
A: $\{B_u\}$ has a zero in $(t, t+a]$,
B: $\{B_u\}$ has a zero in $(t, t+b]$.

Consequently, $\mathbb{P}(A) = \nu(t, a)$ and $\mathbb{P}(B) = \nu(t, b)$. The probability of interest in this problem is then

$$\mathbb{P}(B^c \mid A^c) = \mathbb{P}(B^c A^c) / \mathbb{P}(A^c).$$

Note then that $B^c A^c = B^c$. This might seem counterintuitive at first glance, but you can think about it like this:

If we know that $\{B_u\}$ has no zeros in $(t, t+b]$ (B^c), then we know that it has no zeros in $(t, t+a]$ (A^c). The converse is however not true. Hence $B^c \subset A^c$, which implies that $B^c A^c = B^c$. Now,

$$\begin{aligned}\mathbb{P}(B^c \mid A^c) &= \mathbb{P}(B^c A^c)/\mathbb{P}(A^c) = \mathbb{P}(B^c)/\mathbb{P}(A^c) \\ &= (1 - \mathbb{P}(B))/(1 - \mathbb{P}(A)) \\ &= (1 - \nu(t, b))/(1 - \nu(t, a)).\end{aligned}$$