Exercise 8.2.1

Let $\{B_t : t \geq 0\}$ be a standard Brownian motion with $B_0 = 0$. Furthermore, let $\{M_t : t \geq 0\}$ be the maximumsprocess, i.e. $M_t = \max\{B_u : 0 \leq u \leq t\}$.

a) First note that

$$\mathbb{P}(M_4 \leq 2) = 1 - \mathbb{P}(M_4 > 2) = 1 - 2\mathbb{P}(B_4 > 2).$$

Now,

$$\mathbb{P}(B_4 > 2) = 1 - \mathbb{P}(B_4 \leq 2) = 1 - \mathbb{P}(B_4 - B_0 \leq 2) = 1 - \Phi(1)$$

Hence,

$$\mathbb{P}(M_4 \leq 2) = 1 - 2(1 - \Phi(1)) = 2\Phi(1) - 1 = 0.6826.$$

b) $$\mathbb{P}(M_9 > c) = 2\mathbb{P}(B_9 > c) = 2(1 - \mathbb{P}(B_9 \leq c))$$

$$= 2(1 - \mathbb{P}(B_9 - B_0 \leq c)) = 2(1 - \Phi(c/3)),$$

since $B_9 - B_0 \sim \mathcal{N}(0, 3^2)$. Hence

$$2(1 - \Phi(c/3)) = 0.1 \implies \Phi(c/3) = 0.95$$

$$\implies c/3 = \Phi^{-1}(0.95) = 1.645.$$ Hence $c = 1.645 \cdot 3 = 4.935.$