Exercise 8.1.1

Let $\{B_t : t \geq 0\}$ be a Brownian Motion with $\sigma = 1$ and $B_0 = 1$.

a) $\mathbb{P}(B_4 \leq 3 \mid B_0 = 1) = \mathbb{P}(B_4 - B_0 \leq 2)$

Since $\{B_t\}$ is a Brownian Motion, we have that

$$B_4 - B_0 \sim \mathcal{N}(0, 4)$$

Hence

$$\mathbb{P}(B_4 - B_0 \leq 2) = \Phi(2/\sqrt{4}) = \Phi(1) = 0.8413$$

b)
$$\begin{aligned}
\mathbb{P}(B_9 > c \mid B_0 = 1) &= \mathbb{P}(B_9 - B_0 > c - 1) \\
&= \mathbb{P}((B_9 - B_0)/3 > (c-1)/3) \\
&= 1 - \mathbb{P}((B_9 - B_0)/3 \leq (c-1)/3) \\
&= 1 - \Phi((c-1)/3) = 0.1
\end{aligned}$$

Hence $\Phi((c-1)/3) = 0.9 \implies (c-1)/3 = \Phi^{-1}(0.9) \cong 1.28$

Thus, $c = 3 \cdot 1.28 + 1 = 4.84.$