IMM - DTU

02407 Stochastic Processes
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BFN/bfn

Solution for exercise 28, exercise 1 final exam 5/1-1981

Question 1 The probability is given by the binomial distribution

$$p = \sum_{i=0}^{m} \pi^i (1-\pi)^{n-i}$$

Question 2

$$\mathbf{P} = \begin{bmatrix} q_1 & p_1 & 0 & 0 & \dots & 0 & 0 \\ q_1 & 0 & p_1 & 0 & \dots & 0 & 0 \\ q_1 & 0 & 0 & p_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ q_1 & 0 & 0 & 0 & \dots & 0 & p_1 \\ q_0 & 0 & 0 & 0 & \dots & 0 & p_0 \end{bmatrix}$$

We can assume stationarity under the assumption of a large number of quantities.The stationary probabilities are

$$\pi_i = p_1^i \pi_0, i < k \qquad \pi_k = \left(\frac{q_0(1-p_1^k)}{q_1 p_1^k} + 1 \right)^{-1}$$

Question 3

$$p_1(1-\pi_k) + p_0 \pi_k$$

Question 4

$$\pi_k p_0 (N - n_0)\pi + (1-\pi_k) p_1 (N - n_1)\pi$$

Question 5 $I_k \in NB(k, 1-\pi)$

$$E(I_k) = \frac{k}{1-\pi}$$

Question 6 We always check $n(n_0$ or $n_1)$ items. The expected number of items checked to get n which are working is: $n/(1-\pi)$

With probability q we also have to test the remaining $N-n$. The expected number needed to get $N-n$ working is $(N-n)/(1-\pi)$

The final result is:

$$\pi_k * (n/(1-\pi) + p_0 * (N-n)/(1-\pi)) + (1-\pi_k) * (n/(1-\pi) + p_0(N-n)/(1-\pi)))$$

which can be somewhat reduced.