IMM - DTU

02407 Stochastic Processes
2008-11-17
BFN/bfn

Solution for exercise 11, ex.2 final examination 16/12-1991

Question 1 The problem is to estimate the expected number of events in an ordinary renewal process. With hyperexponentially distributed times the problem is equivalent to the example 1.10 in the MAP-note. From this solution we have

$$H_0(t) = \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} - \frac{p_1 p_2 (\frac{1}{\lambda_1} - \frac{1}{\lambda_2})^2}{\mu^2} e^{-t(p_2\lambda_1 + p_1\lambda_2)}$$

We find $\mu = \frac{17}{20} \cdot 2 + \frac{3}{20} \cdot 6 = \frac{13}{5}$. $\sigma^2 = 2(\frac{17}{20} \cdot 4 + \frac{3}{20} \cdot 36) - \left(\frac{13}{5}\right)^2 = \frac{88}{5} - \frac{169}{25} = \frac{271}{25}$ And from this we find $H_0(10) = 4.11$.

Question 2 The president, who is hired immediately before $t = 0$ is not included in the three presidents. The easiest way is to give either an expression for the Laplacetransformation or a phase type representation. The Laplace transformation $H(s)$

$$\tilde{H}(s) = \left(\frac{\frac{1}{2} \cdot \frac{17}{20}}{s + \frac{1}{2}} + \frac{\frac{1}{6} \cdot \frac{3}{20}}{s + \frac{1}{6}}\right)^3$$

The phase type representation $(\vec{\alpha}, \mathbf{T})$ med $\vec{\alpha} = \left(\frac{17}{20}, \frac{3}{20}, 0, 0, 0, 0\right)$

$$\mathbf{T} = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{17}{40} & \frac{3}{40} & 0 & 0 \\ 0 & -\frac{1}{6} & \frac{17}{120} & \frac{3}{120} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & \frac{17}{40} & \frac{3}{40} \\ 0 & 0 & 0 & -\frac{1}{6} & \frac{17}{120} & \frac{3}{120} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{6} \end{bmatrix}$$

Question 3 Formula (126) p.34 chapter 1 in Wolff is used. Alternatively the result of exercise 3.7.11 is applied for exercise 3.7.13 to obtain $V(S) = E(N)V(X) + V(N)E(X)^2$. Then apply this result to get

$$E\{Expense\} = p \cdot 200.000 - (1-p) \cdot 25.000 = \frac{3}{20} \cdot 200.000 - \frac{17}{20} \cdot 25.000 = 8.750$$

$$V\{Expense\} = p \cdot 200.000^2 + (1-p) \cdot 25.000^2 - 8.750^2 = (80, 341.07)^2$$

$$E\{Totalexpenses\} = 8,750 \cdot 4,11 = 35,962.5$$

$$V\{Totalexpenses\} = 8,750^2 \cdot 4.85 + 4.11 \cdot 80,341.07^2 = 164,012.5^2$$

Question 4 We use the note on Markov renewal processes.

$$\tilde{\mathbf{A}}(s) = \begin{bmatrix} \frac{17}{20} & \frac{3}{20} \\ \frac{17}{20} & \frac{3}{20} \end{bmatrix} \begin{bmatrix} \frac{\frac{1}{2}}{s+\frac{1}{2}} & 0 \\ 0 & \frac{\frac{1}{6}}{s+\frac{1}{6}} \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left(\frac{17}{20}, \frac{3}{20} \right) \begin{bmatrix} \frac{\frac{1}{2}}{s+\frac{1}{2}} & 0 \\ 0 & \frac{\frac{1}{6}}{s+\frac{1}{6}} \end{bmatrix}$$

$$s\tilde{\mathbf{Q}}(s) = \left(\mathbf{I} - \tilde{\mathbf{A}}(s)\right)^{-1} \left(\mathbf{P} - \tilde{\mathbf{A}}(s)\right)$$

$$\mathbf{P} = \begin{bmatrix} \frac{17}{20} & \frac{3}{20} \\ \frac{17}{20} & \frac{3}{20} \end{bmatrix}$$

Or

$$s\tilde{\mathbf{Q}}(s) = \left(\mathbf{I} - \tilde{\mathbf{A}}(s)\right)^{-1} \mathbf{P} \left(\mathbf{I} - \begin{bmatrix} \frac{\frac{1}{2}}{s+\frac{1}{2}} & 0 \\ 0 & \frac{\frac{1}{6}}{s+\frac{1}{6}} \end{bmatrix} \right)$$

Alternatively the process can be regarded as a phase type renewal process or as a Markov chain in continous time. In that case we can express the probabilities in the time domain by

$$Q(t) = \boldsymbol{e} \left(\frac{17}{20}, \frac{3}{20} \right) \mathrm{e}^{\begin{bmatrix} -\frac{3}{40} & \frac{3}{40} \\ \frac{17}{120} & -\frac{17}{120} \end{bmatrix} t}$$