IMM - DTU

02407 Stochastic Processes
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BFN/bfn

Solution for exercise 19, exercise 3 final exam 2/1-1995

Question 1 From the two state Markov chain we find

$$\frac{\omega_2}{\omega_1+\omega_2}$$

Question 2

$$\lambda\frac{\omega_2}{\omega_1+\omega_2}$$

Question 3 The time between two arrivals can be described by a phase type distribution (time to absorption in finite Markov chain) with representation $\vec{\alpha}=(1,0)$ and:

$$T=\left[\begin{array}{cc} -(\omega_1+\lambda) & \omega_1 \\ o\omega_2 & -\omega_2 \end{array}\right]$$

The process is a renewal process.

Question 4

$$\left(\frac{\omega_2}{\omega_1+\omega_2}\right)^2$$

By independence

Question 5 As the processes are independent, the number og processes i state i at a random point in time be binominally distributed.

$$\left(\begin{array}{c} N \\ i \end{array}\right)\left(\frac{\omega_2}{\omega_1+\omega_2}\right)^i\left(\frac{\omega_1}{\omega_1+\omega_2}\right)^{N-i}$$

Question 6 As the processes are independent the mean and variance of the total number in the time interval $]0;t]$ can be found as the sum of the values for the separaate processes. So the requested proportion for N processes is the same as the proportion for a single process. The mean number of events in the stationary renewalprocess in the interval till t is $\frac{\lambda\omega_2 t}{\omega_1+\omega_2}$, and we find

$$1+\frac{2\lambda\omega_1}{(\omega_1+\omega_2)^2}\left(1-\frac{1}{(\omega_1+\omega_2)t}\left(1-e^{-(\omega_1+\omega_2)t}\right)\right)$$