IMM - DTU

02407 Stochastic Processes
2012-10-30
BFN/bfn

Solution exercise 2

Question 1 The residual life time is also phase type distributed with representation $(\vec{\pi}, \mathbf{T})$. The vector $\vec{\pi}$ is the solution to

$$\vec{\pi}(\mathbf{T} + \vec{T^0}\vec{\alpha}) = \vec{0}$$

In this case we get

$$\vec{\pi}\left(\left[\begin{array}{cc} -\lambda & \lambda \\ 0 & -\lambda \end{array}\right] + \left[\begin{array}{c} 0 \\ \lambda \end{array}\right](1,0)\right) = (0,0)$$

$$\vec{\pi}\left[\begin{array}{cc} -\lambda & \lambda \\ \lambda & -\lambda \end{array}\right] = (0,0)$$

and finally $\vec{\pi} = (\frac{1}{2}, \frac{1}{2})$. The interpretation is that the residual life time is exponentially distributed with probability $\frac{1}{2}$ and Erlang-2 distributed with probability $\frac{1}{2}$

$$F(t) = \frac{1}{2}\left(1 - e^{-\lambda t}\right) + \frac{1}{2}\left(1 - e^{-\lambda t} - \lambda t e^{-\lambda t}\right) =$$

$$F(t) = 1 - e^{-\lambda t} - \frac{\lambda t}{2} e^{-\lambda t}$$

Another approach woild be to calculate $\vec{\alpha}e^{\mathbf{T}t}\vec{e}$ or the Laplace transform $(1,0)(s\mathbf{I} - \mathbf{T})^{-1}\vec{T^0}$.

Question 2 The cumulative distribution function was found in question 1. The probability in question is $1 - F(1) = 0.5518$. The unit Thus, it is decided to exchange that piece of equipment with a new piece.

Question 3 The same formulae apply for the second kind of equipment. Alternatively, one could identify the process as a renewal process with H_2 distributed interarrival times.

$$\vec{\pi} = \left(\frac{\frac{p_1}{\lambda_1}}{\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}}, \frac{\frac{p_2}{\lambda_2}}{\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}}\right)$$

and:

$$F(t) = 1 - \pi_1 e^{-\lambda_1 t} - \pi_2 e^{-\lambda_2 t}$$

The probability in question is 0.8089, in this case, and the equipment is not exchanged.

Question 4 The expression found above for $e^{\mathbf{T}t}$ can be reused. The distribution for the first type is Erlang-2 distributed while it is H_2 distributed for the second type. The two probabilities in question are 0.7358 and 0.1391. It should be noticed that the mean value of the two distributions are equal (=1).