

## Solution for exercise 19, exercise 3 final exam 2/1-1995

Question 1 From the two state Markov chain we find

$$\frac{\omega_2}{\omega_1 + \omega_2}$$

Question 2

$$\lambda \frac{\omega_2}{\omega_1 + \omega_2}$$

Question 3 The time between two arrivals can be described by a phase type distribution (time to absorption in finite Markov chain) with representation  $\vec{\alpha} = (1, 0)$  and:

$$T = \begin{bmatrix} -(\omega_1 + \lambda) & \omega_1 \\ \omega_2 & -\omega_2 \end{bmatrix}$$

The process is a renewal process.

Question 4

$$\left( \frac{\omega_2}{\omega_1 + \omega_2} \right)^2$$

By independence

Question 5 As the processes are independent, the number of processes in state  $i$  at a random point in time be binomially distributed.

$$\binom{N}{i} \left( \frac{\omega_2}{\omega_1 + \omega_2} \right)^i \left( \frac{\omega_1}{\omega_1 + \omega_2} \right)^{N-i}$$

Question 6 As the processes are independent the mean and variance of the total number in the time interval  $]0; t]$  can be found as the sum of the values for the separate processes. So the requested proportion for  $N$  processes is the same as the proportion for a single process. The mean number of events in the stationary renewal process in the interval till  $t$  is  $\frac{\lambda \omega_2 t}{\omega_1 + \omega_2}$ , and we find

$$1 + \frac{2\lambda\omega_1}{(\omega_1 + \omega_2)^2} \left( 1 - \frac{1}{(\omega_1 + \omega_2)t} (1 - e^{-(\omega_1 + \omega_2)t}) \right)$$