IMM - DTU

02407 Stochastic Processes
2009-12-20
BFN/bfn

Solution for exercise 14, ex. 1 final exam 9/1-1993

Question 1 With the given assumptions the lifetime can be described by a Markov chain with 3 transient and one absorbinbg state

$$\mathbf{Q} = \begin{bmatrix} -0.01 & 0.0095 & 0.0005 & 0 \\ 0.08 & -0.1 & 0.02 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The lifetime is the time to absorption with $\vec{P}(0) = (1, 0, 0, 0)$. The distribution can be described as a phasetype distribution where $\vec{\alpha} = (1, 0, 0)$ and:

$$\mathbf{T} = \begin{bmatrix} -0.01 & 0.0095 & 0.0005 \\ 0.08 & -0.1 & 0.02 \\ 0 & 0 & -1 \end{bmatrix}$$

Question 2 The sojourn time is exponentially distributed, the requested probality is e^{-2}.

Question 3 Now we have a Markov chain in continous time, derived from the phase type distribution:

$$\mathbf{Q}^{\star} = \mathbf{T} + \vec{T}^{0}\vec{\alpha} = \begin{bmatrix} -0.01 & 0.0095 & 0.0005 \\ 0.08 & -0.1 & 0.02 \\ 1 & 0 & -1 \end{bmatrix}$$

The stationary probabilities for the states in this distribution express the expected time, the p.t.working device will be in respectively the OK, unreliable and critical state. As solution of

$$\vec{\pi}\mathbf{Q}^{\star} = \vec{0}$$

We find $\vec{\pi} = \left(\frac{10000}{10974}, \frac{950}{10974}, \frac{24}{10974}\right) = (0,9112; 0,0866; 0,0022)$.

Question 4 As $\mathbf{T}$ is nonsingular we can multiply $\mathbf{T}^{-1}$. Thus

$$\vec{\pi}\left(\mathbf{T} + \vec{T}^{0}\vec{\alpha}\right)\mathbf{T}^{-1} = \vec{0}\mathbf{T}^{-1} \Leftrightarrow$$

$$\vec{\pi} = -\vec{\pi}\vec{T}^{0}\vec{\alpha}\mathbf{T}^{-1}$$

In this equation we can multiply the vector $\vec{e}$ på from right and obtain å

$$1 = (\vec{\pi}\vec{T}^{0})(-\vec{\alpha}\mathbf{T}^{-1}\vec{e}) \Leftrightarrow$$

$$(\vec{\pi}\vec{T}^{0})^{-1} = (-\vec{\alpha}\mathbf{T}^{-1}\vec{e})$$

The expression on the right side in the equation is μ, and this answers the question.

Question 5 The results from questions 3 and 4 are used to find the mean life time. The mean life time is thus given by

$$\mu = \left((0,9112; 0,0866; 0,0022) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)^{-1} = 457,25$$