IMM - DTU

04141 Stochastic Processes
1998-12-14
BFN/bfn

Solution for exercise 13, ex. 4 final examination 16/12-1991

Question 1 The distribution is a phase type distribution with $\vec{\alpha} = \left(\frac{3}{4}, \frac{1}{4}\right)$.

$$\mathbf{T} = \begin{bmatrix} -1 & 0.9 \\ \frac{12}{55} & -\frac{12}{55} \end{bmatrix}$$

The mean is:

$$\mu = \vec{\alpha}(-\mathbf{T})^{-1}\vec{e} = \frac{2515}{48} = 52.40$$

$$E(X^2) = \left(\frac{3}{4}, \frac{1}{4}\right) \cdot 2 \begin{bmatrix} 512.5 & 2303.125 \\ 558.33 & 2513.19444 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 5759.20139$$

$$\sigma^2 = 3013.878$$

Question 2 We use the Laplace transform:

$$\tilde{H}(s) = \left(\frac{3}{4}, \frac{1}{4}\right) \frac{1}{\left(s + \frac{6}{5}\right)\left(s + \frac{1}{55}\right)} \begin{bmatrix} s + \frac{12}{55} & 0.9 \\ \frac{12}{55} & s + 1 \end{bmatrix} \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} = \frac{5}{104}\frac{\frac{6}{5}}{s + \frac{6}{5}} + \frac{99}{104}\frac{\frac{1}{55}}{s + \frac{1}{55}}$$

$$f(t) = \frac{5}{104}\frac{6}{5}e^{-\frac{6}{5}t} + \frac{99}{104}\frac{1}{55}e^{-\frac{1}{55}t}$$

Question 3

$$\mathbf{Q}^\star = \mathbf{T} + \vec{T}^0\vec{\alpha} = \begin{bmatrix} -\frac{37}{40} & \frac{37}{40} \\ \frac{12}{55} & -\frac{12}{55} \end{bmatrix}$$

$$\pi_1\frac{37}{40} = \pi_2\frac{12}{55}$$

$$\pi_1 = \frac{96}{503} \qquad \pi_2 = \frac{407}{503}$$

Question 4 The distribution is phase type with $(\vec{\pi}, \mathbf{T})$.

$$\mu_1^\star = \pi(-\mathbf{T})^{-1}\vec{e} = \left(\frac{96}{503}, \frac{407}{503}\right) \begin{pmatrix} 51.25 \\ 55.833 \end{pmatrix} = 54.959$$

$$\mu_2^\star = 2\pi(-\mathbf{T})^{-2}\vec{e} = \left(\frac{96}{503}, \frac{407}{503}\right) \begin{pmatrix} 2815.625 \\ 3071.52777 \end{pmatrix} = 6045.37497$$