IMM - DTU

02407 Stochastic Processes
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DAME/dame

Solution for exercise 9.3.2 in Karlin and Pinsky

We analyse two different scenarios. We choose the que length as criterium to choose the methode we prefer. This is valid since $W = \frac{L}{\lambda}$ and the arrival rate in both cases is the same

In case A we choose L_A for the wainting line and use as parameters $\tau_A^2 = 0,2$ and $v_A = 0,5$ in case B we choose L_B aws the que length and we have $\tau_B^2 = 0,9$ and $v = 0,4$. Using equation 9.35 on p.464 in Karlin and Pinsky we get:

$$\begin{aligned}
L_A &= \frac{2\lambda v + \lambda^2\tau^2 - (\lambda v)^2}{2(1-\lambda v)} \\
&= \frac{2 \cdot 1 \cdot 0,5 + 1^2 \cdot 0,2 - (0,5)^2}{2(1 - 1 \cdot O,5)} \\
&= \frac{1 + 0,2 - 0,25}{1} \\
&= 0,95 \\
L_B &= \frac{2 \cdot 1 \cdot 0,4 + 1^2 \cdot 0,9 - (0,4)^2}{2(1 - 1 \cdot O,4)} \\
&= \frac{0,8 + 0,9 - 0,16}{1,2} \\
&= \frac{1,54}{1,2} > 0,95
\end{aligned}$$

Therefore we chosse procedure A

Since the system only is stable if $\lambda \cdot v < 1$ we know it becomes unstable for system A before system B and therefore it would be wise to switch.