IMM - DTU

02407 Stochastic Processes
2011-12-05
DAME/dame

Solution for exercise 9.3.1 in Karlin and Pinsky

We have to calculated $P(X(t) = x, Y(t) = y)$. Let $Z(t)$ be the number of arrivals until time t. then we know

$$P(X(t) = x, Y(t) = y) = P(X(t) = x, Z(t) - [X(t) - X(0)] = y)$$

Without loss of generality we can choose $X(0) = 0$ and get

$$\begin{aligned}
P(X(t) = x, Y(t) = y) &= P(X(t) = x, Z(t) - [X(t) - X(0)] = y) \\
&= P(Z(t) - X(t) = y | X(t) = x) P(X(t) = x) \\
&= P(Z(t) = x + y) P(X(t) = x) \\
&= \frac{(\lambda t)^{x+y}}{(y+x)!} e^{-\lambda t} \cdot \frac{\mu(A_t)^x e^{\mu(A_t)}}{x!}
\end{aligned}$$

The result for $P(X(t) = x)$ comes from p.466 in Karlin and Pinsky