IMM - DTU

02407 Stochastic Processes
2011-12-05
DAME/dame

Solution for exercise 9.2.7 in Karlin and Pinsky

0.1 a)

Using equation 6.24 we get for the forward equations:

$$\begin{aligned} p'_{i0}(t) &= -\lambda p_{i0}(t) + \mu p_{i1}(t) \\ p'_{ij}(t) &= \lambda p_{i,j-1}(t) - (\lambda + j\mu) p_{ij}(t) + (j+1)\mu p_{i,j+1}(t) \end{aligned}$$

0.2 b)

$$
\begin{aligned}
M'(t) &= \sum_{j=0}^{\infty} j \cdot p'_{i,j} \\
&= \sum_{j=1}^{\infty} j[\lambda p_{i,j-1}(t) - \lambda p_{ij}(t) - j\mu p_{ij}(t) + (j+1)\mu p_{i,j+1}(t)] \\
&= \sum_{j=0}^{\infty} (j+1)\lambda p_{ij}(t) - \sum_{j=1}^{\infty} j\lambda p_{ij}(t) - \sum_{j=1}^{\infty} j^2 \mu p_{ij}(t) + \sum_{j=1}^{\infty} j(j+1)\mu p_{i,j+1}(t) \\
&= \lambda - \sum_{j=1}^{\infty} j^2 \mu p_{ij}(t) + \sum_{j=1}^{\infty} j^2 \mu p_{ij+1}(t) + \sum_{j=1}^{\infty} j\mu p_{ij+1}(t) \\
&= \lambda - \sum_{j=1}^{\infty} J^2 \mu p_{ij}(t) + \sum_{j=2}^{\infty} j^2 \mu p i_{ij}(t) - 2\sum_{j=2}^{\infty} j\mu p i_{ij}(t) \\
&+ \sum_{j=2}^{\infty} \mu p_{ij}(t) + \sum_{j=2}^{\infty} j\mu p_{Iij} - \sum_{j=2}^{\infty} \mu p_{ij}(t) \\
&= \lambda - \mu p_{i1} - \sum_{j=2}^{\infty} j\mu p_{ij}(t) \\
&= \lambda - \sum_{j=1}^{\infty} j\mu p_{ij}(t) \\
&= \lambda - \mu M(t)
\end{aligned}
$$

0.3 c)

We have to solve the equation $M'(t) + \mu M(t) = \lambda$. We can use standart analys results and get

$$M(t) = \lambda + e^{-\mu t}$$