

## Solution for exercise 9.2.6 in Karlin and Pinsky

We have to find the probability that under stationarity  $P(X_\pi(t) > c) < 0.001$ .

We know:

$$\begin{aligned} P(X_\pi(t) > c) &= 1 - P(X_\pi(t) \geq c) \\ &= 1 - \sum_{k=0}^c \pi_k \\ &= 1 - \sum_{k=0}^c (1 - \rho) \rho^k \end{aligned}$$

Using this we can calculate  $c$

$$\begin{aligned} P(X_\pi(t) > c) &< 0,001 \\ \Leftrightarrow 1 - \sum_{k=0}^c (1 - \rho) \rho^k &< 0,001 \\ \Leftrightarrow \sum_{k=0}^c (1 - \rho) \rho^k &> 0,999 \\ \Leftrightarrow 1 - \rho^{c+1} &> 0,999 \\ \Leftrightarrow \rho^{c+1} &< 0,001 \\ \Leftrightarrow c &< \frac{\ln(0,001)}{\ln(\rho)} - 1 \end{aligned}$$