IMM - DTU

02407 Stochastic Processes
2022-10-10
DAME/dame

Solution for exercise 9.2.4 in Karlin and Pinsky

0.1 a)

instead of stating the death and birht parameters, we state the transition matrix

$$Q = \begin{pmatrix} -\lambda & \lambda & 0 & 0 \\ \mu & -(\lambda+\mu) & \lambda & 0 \\ 0 & \mu & -(\lambda+\mu) & \lambda \\ 0 & 0 & \mu & \mu \end{pmatrix}$$

0.2 b)

$$\begin{aligned} \pi_1 &= \frac{\lambda}{\mu}\pi_0 \\ \pi_2 &= \frac{\lambda}{\mu}\pi_1 = (\frac{\lambda}{\mu})^2\pi_0 \\ \pi_3 &= \frac{\lambda}{\mu}\pi_2 = (\frac{\lambda}{\mu})^3\pi_0 \\ \Rightarrow \quad \pi_0 &= \frac{1}{1+\frac{\lambda}{\mu}+(\frac{\lambda}{\mu})^2+(\frac{\lambda}{\mu})^3} \\ &= \frac{\mu^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3} \end{aligned}$$

π_0 is the fraction of time where the system is empty!

0.3 c)

A customer is lost when 3 customers are present and a new one arrives. Therefore the fraction of customers lost is $\lambda\pi_3$