IMM - DTU

02407 Stochastic Processes
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# Solution for exercise 8.4.6 in Karlin and Pinsky

We have to calculate $P(\max_{t\geq 0}\ Z(t) > 2x; t \geq 0 | Z(0) = x)$. If we define $T = T_{A,B} = min\left\{t \geq 0; \frac{Z(t)}{Z(0)} = A \text{ or } \frac{Z(t)}{Z(0)} = B\right\}$ and chosse $A = 0$ and $B = 2$ than we can use Theorem 8.3

$$\begin{aligned} P(\max_{t\geq 0}\ Z(t) > 2x; t \geq 0 | Z(0) = x) &= P\left(\frac{Z(T)}{Z(0)} = B\right) \\ &= \frac{1 - A^{1-2\alpha/\sigma^2}}{B^{1-2\alpha/\sigma^2} - A^{1-2\alpha/\sigma^2}} \\ &= \frac{1}{2} \end{aligned}$$