

Solution for exercise 8.4.5 in Karlin and Pinsky

We can use Theorem 8.1 Define $T_{0\ b} = \min \{t \geq 0; X(t) = 0 \text{ or } X(t) = b\}$ and we get:

$$P(X(T_{0\ b}) = b | X(0) = x) = \frac{e^{-2\mu x/\sigma^2} - 1}{e^{-2\mu b/\sigma^2} - 1}$$

This is the probability that the absorbed Brownian motion ever reaches the height $b > x$