IMM - DTU

02407 Stochastic Processes
2012-11-20
DAME/dame

Solution for exercise 8.4.4 in Karlin and Pinsky

We have to look at

$$\begin{aligned} X'(t) &= X(t) - \frac{1}{2}t(\mu_0 + \mu_1) = \mu t + \sigma B(t) - \frac{1}{2}t(\mu_0 = \mu_1) \\ &= \sigma B(t) - \frac{1}{2}t(\mu_0 - \mu) - \frac{1}{2}t(\mu_1 - \mu) \\ &= \begin{cases} \sigma B(t) - \frac{1}{2}t(\mu_1 - \mu_0) \text{ if } \mu = \mu_0 \\ \sigma B(t) + \frac{1}{2}t(\mu_1 - \mu_0) \text{ if } \mu = \mu_1 \end{cases} \end{aligned}$$

If we choose $\delta = (\mu_1 - \mu_0)$ we can apply the results from the example on page 422 in Karlin and Pinsky. if $X'(t)$ has drift $\frac{1}{2}\delta$ we know that $X(t)$ has drift μ_1 and vice versa. If we observe $X'(t)$ and it hits $b = \frac{1}{\mu_1 - \mu_0} log\left(\frac{1-\beta}{\alpha}\right)$ before it hits $a = -\frac{1}{\mu_1 - \mu_0} log\left(\frac{1-\alpha}{\beta}\right)$ then the original process has μ_1. If the process hits first a then the original process has drift μ_0