IMM - DTU

02407 Stochastic Processes
2012-11-20
DAME/dame

Solution for exercise 8.4.1 in Karlin and Pinsky

$$P(\max_t \ B(t) > a + bt) = P(\max \ (B(t) - bt) > a)$$

Define $X(t) := B(t) - b \cdot t$ than it is a Brownian motion with drift. Furthermore we know $\sigma = 1$ and $\mu = -b$ and with this we can use theorem 8.1 in Karlin and Pinsky on page 420. Define $T_{\tilde{b}a} = min \left\{ t \geq 0; \ X(t) = a \text{ or } X(t) = \tilde{b} \right\}$

$$\begin{aligned} P(\max_t \ B(t) > a + bt) &= P(X(t) > a) \\ &= \lim_{\tilde{b} \to -\infty} P(X(T_{\tilde{b}a}) = a) \\ &= \lim_{\tilde{b} \to -\infty} \frac{1 - e^{2b\tilde{b}/\sigma^2}}{e^{2ba/\sigma^2} - e^{2b\tilde{b}/\sigma^2}} \\ &= e^{-2b \cdot a} \end{aligned}$$