IMM - DTU

02407 Stochastic Processes
2011-11-14
DAME/dame

Solution for exercise 8.3.6 in Karlin and Pinsky

Define $T := \{\min\{t \geq 0,\ A(t) \notin (0, z)\}\}$. Since we have a continous function we know $P(M > z|A(0) = x) = P(M \geq z|A(0) = x)$ but $P(M \geq z|A(0) = x) = P(A(T) \geq z|A(0) = x)$.
Until T behaves the absorbed Brownian motion for sure like a brownien motion and we know:

$$\begin{aligned} E[A(T) - A(0)|A(0)] &= 0 \\ \Leftrightarrow\ E[A(T)|A(0)] &= x \end{aligned}$$

We can also calculate the expectation directly.

$$\begin{aligned} E[A(T)|A(0)] &= z \cdot P(A(T) \geq z|A(0) = x) + 0 \cdot P(A(T) = 0|A(0) = x) \\ &= z \cdot P(A(T) \geq z|A(0) = x) \\ \Rightarrow\ x &= z \cdot P(A(T) \geq z|A(0) = x) \\ \Leftrightarrow\ \frac{x}{z} &= P(A(T) \geq z|A(0) = x) = (M > z|A(0) = x) \end{aligned}$$