IMM - DTU

02407 Stochastic Processes
2011-11-8
DAME/dame

Solution for exercise 8.1.2 in Karlin and Pinsky

We recall for a radom variable X and a function $g(\cdot)$ we can calculate $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$

$$\begin{aligned}
E[e^{\lambda B(t)}] &= \int_{\infty}^{\infty} e^{\lambda x} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2t}x^2} dx \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2t}x^2+\lambda x} dx \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2t}(x^2-2\lambda t x)} dx \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}[(x-\lambda t)^2-(\lambda t)^2]} dx \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2t}(x-\lambda t)^2} dx \cdot e^{\frac{1}{2t}(\lambda t)^2} \\
&= e^{\lambda^2 \frac{t}{2}}
\end{aligned}$$