IMM - DTU

02407 Stochastic Processes
2011-11-8
DAME/dame

Solution for exercise 8.1.1 in Karlin and Pinsky

Define $T = T_{a,b} = \min(t \geq 0\ ,\ B(t) = -a\ or\ B(t) = b)$. We know $E[\xi_i] = 0$ and $Var[\xi_i] = 1$ therefore we can try to aproximate $B(t)$ with $\frac{S_{[nt]}}{\sqrt{n}}$.

$$\begin{aligned} E[T] &= E[\lim_{n\to\infty} \min(t \geq 0\ ,\ \frac{S_{[nt]}}{\sqrt{n}} = -a\ or\ \frac{S_{[nt]}}{\sqrt{n}}) = b)] \\ &= E[\lim_{n\to\infty} \min(t \geq 0\ ,\ S_{[nt]} = -a\sqrt{n}\ or\ S_{[nt]}) = b\sqrt{n})] \end{aligned}$$

Using the result from section 3.5.3 this leaves us with solving:

$$\begin{aligned} \min_t\ [nt] &= abn \\ \Leftrightarrow t &= ab \end{aligned}$$

and with this we get; E[T]=ab