IMM - DTU

02407 Stochastic Processes
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DAME/dame

Solution for exercise 7.3.5 in Karlin and Pinsky

Since we are facing a Poisson process we have to consider the residual lifetime as well as the current life time.

$$\begin{aligned}
P(D_t \leq x) &= P\left(\min(\gamma_t, \delta_t) \leq x\right) \\
&= 1 - P\left(\min(\gamma_t, \delta_t) > x\right) \\
&= 1 - P(\gamma_t > x)P(\delta_t > x) \\
&= \begin{cases} 1 - e^{-\lambda x}e^{-\lambda x} \text{ if } x < t \\ 1 - e^{-\lambda x}P(\text{no bird on the wire until t}) = 1 - e^{-\lambda x}e^{-\lambda t} \text{ if } t < x \end{cases}
\end{aligned}$$

we then get

$$f_{D_t}(x) = \frac{dF_{D_t}(x)}{dx} = \begin{cases} 2\lambda e^{-2\lambda x} \text{ if } x < t \\ \lambda e^{-\lambda x}e^{-\lambda t} \text{ if } x > t \end{cases}$$