IMM - DTU

02407 Stochastic Processes
2012-10-08
DAME/dame

Solution for exercise 6.5.2 in Karlin and Pinsky

Using the definition on page 317 we can calculate ρ_i

$$\rho_0 = 1 \quad \rho_1 = 4 \quad \rho_2 = 6$$
$$\rho_3 = 4 \quad \rho_4 = \frac{24}{24} = 1$$

0.1 a)

and with equation (6.43) we get for $u_i = \lim_{t\to\infty} P(X_t = 0|X_0 = i)$ and $u_5 = 0$

$$\begin{aligned} & u_5 - u_1 = (u_1 - 1)\sum_{i=1}^{4} \rho_i \\ \Leftrightarrow \quad & -u_1 = (u_1 - 1) \cdot 15 \\ \Leftrightarrow \quad & 16 \cdot u_1 = 15 \\ \Leftrightarrow \quad & u_1 = \frac{15}{16} \end{aligned}$$

and

$$\begin{aligned} & u_2 - u_1 = (u_1 - 1)\rho_1 \\ \Leftrightarrow \quad & u_2 - \frac{15}{16} = -\frac{1}{16} \cdot 4 \\ \Leftrightarrow \quad & u_2 = \frac{15}{16} - \frac{4}{16} = \frac{11}{16} \end{aligned}$$

0.2 b)

Let w_i be the mean absorbtion time starting from state i. Do to the symmetry of the system, we know $w_1 = w_4$, $w_2 = w_3$ and $w_0 = w_5 = 0$. Equation 6.47

page 319 is valide and we get:

$$\begin{aligned}
&\frac{1}{\rho_4}(w_4 - w_5) = \sum_{i=1}^{4} \frac{1}{\lambda_i \rho_i} - w_1 \\
\Leftrightarrow\ \ & w_4 = \frac{2}{3} - w_1 \\
\Leftrightarrow\ \ & w_1 + w_4 = \frac{2}{3} \\
\Rightarrow\ \ & w_1 = \frac{1}{3}\ and\ w_4 = \frac{1}{3}
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{\rho_3}(w_3 - w_4) = \sum_{i=1}^{3} \frac{1}{\lambda_i \rho_i} - w_1 \\
\Leftrightarrow\ \ & \frac{1}{4}(w_3 - \frac{1}{3}) = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} - \frac{1}{3} \\
\Leftrightarrow\ \ & w_3 = 1 + \frac{1}{3} + \frac{1}{3} - 1 \\
\Leftrightarrow\ \ & w_3 = \frac{2}{3} \\
\Rightarrow\ \ & w_2 = \frac{2}{3}
\end{aligned}$$