IMM - DTU

02407 Stochastic Processes
2012-10-08
DAME/dame

Solution for exercise 6.4.4 in Karlin and Pinsky

0.1 a)

The brith and death parameter depend on the number of couples at time t.

$$\begin{aligned}\lambda_k &= (6-k)\alpha \\ \mu_k &= k\beta\end{aligned}$$

0.2 b)

We can calculate:

$$\theta_i = \begin{cases} 1 \; i = 0 \\ \binom{6}{i} \cdot (\frac{\alpha}{\beta})^i \; 1 \leq i \leq 6 \end{cases}$$

Using the binomial formula we get

$$\sum_{i=0}^{6} \binom{6}{i} (\frac{\alpha}{\beta})^i = (1 + \frac{\alpha}{\beta})^6$$

and therefore $\pi_0 = 1/(1+\frac{\alpha}{\beta})^6 = (\frac{\beta}{\alpha+\beta})^6$. Applying this to equation 6.37 in KP we finally get

$$\begin{aligned}\pi_i &= \binom{6}{i}(\frac{\alpha}{\beta})^i(\frac{\beta}{\alpha+\beta})^6 \\ &= \binom{6}{i}(\frac{\alpha}{\beta})^i\left(\frac{\beta}{\alpha+\beta}\frac{\alpha+\beta}{\beta}\right)^i(\frac{\beta}{\alpha+\beta})^6 \\ &= \binom{6}{i}(\frac{\alpha}{\alpha+\beta})^i(\frac{\beta}{\alpha+\beta})^{6-i}\end{aligned}$$

the binomial distribution.