IMM - DTU

02407 Stochastic Processes
2012-10-05
DAME/dame

Solution for exercise 6.3.4 in Karlin and Pinsky

We know $V(0) = 0$. A stop and go traveler can either travel (being in state 1) or stop (being in state). We then know

$$\begin{aligned} P(V(t) = 1|V(0) = 0) &= P_{01}(t) \\ P(V(t) = 0|V(0) = 0) &= P_{00}(t) \end{aligned}$$

$$E[V(t)] = 1 \cdot P(V(t) = 1|V(0) = 0) = \pi - \pi e^{-\tau \cdot t}$$

We are allowed to use

$$E[S(t)] = \int_0^t E[V(u)]du,$$

and with that we get

$$\begin{aligned} E[S(t)] &= \int_0^t E[V(u)]du \\ &= \int_0^t E[p_{01}(u)]du \\ &= \int_0^t \pi - \pi e^{\tau \cdot u} du \\ &= [\pi \cdot u + \frac{\pi}{\tau} e^{-\tau \cdot u}]_0^t \\ &= \pi \cdot t + \alpha \cdot e^{-(\alpha+\beta) \cdot t} \end{aligned}$$