Solution for exercise 6.3.4 in Karlin and Pinsky

We know $V(0) = 0$. A stop and go traveler can either travel (being in state 1) or stop (being in state 0). We then know

$$P(V(t) = 1|V(0) = 0) = P_{01}(t)$$
$$P(V(t) = 0|V(0) = 0) = P_{00}(t)$$

$$E[V(t)] = 1 \cdot P(V(t) = 1|V(0) = 0) = \pi - \pi e^{-\tau \cdot t}$$

We are allowed to use

$$E[S(t)] = \int_0^t E[V(u)]du,$$

and with that we get

$$E[S(t)] = \int_0^t E[V(u)]du$$
$$= \int_0^t E[p_{01}(u)]du$$
$$= \int_0^t \pi - \pi e^{-\tau \cdot u}du$$
$$= [\pi \cdot u + \frac{\pi}{\tau} e^{-\tau \cdot u}]_0^t$$
$$= \pi \cdot t + \alpha \cdot e^{-(\alpha + \beta) \cdot t}$$