IMM - DTU

02407 Stochastic Processes
2011-9-27
DAME/dame

Solution for exercise 6.3.3 in Karlin and Pinsky

Since we start with the limiting distribution we know

$$E[V(l)] = 0 \cdot (1-\pi) + 1 \cdot \pi \; \forall \; l \geq 0$$

Furthermore

$$\begin{aligned}
E[V(s)V(t)] &= E[V(s)V(t)|V(s)=0]P(V(s)=0) + E[V(s)V(t)|V[s)=1]P(V(s)=1) \\
&= 0 \cdot P(V(s)=0) + [0 \cdot P(V(t-s)=0|V(s)=1) \\
&+ 1 \cdot P(V(t-s)=1|V(s)=1)]P(V(s)=1) \\
&= 1 \cdot (1 - P(V(t-s)=1|V(s)=0))P(V(s)=1) \\
&= (1-p_{10}(t-s))\pi \\
&= \pi - \pi p_{10}(t-s)
\end{aligned}$$

To calculate the covariance

$$\begin{aligned}
Cov[V(s)V(t)] &= E[V(s)V(t)] - E[V(t)]E[V(s)] \\
&= \pi - \pi p_{10}(t-s) - \pi\pi \\
&= \pi - \pi(1-\pi) + \pi(1-\pi)e^{-\tau(t-s)} - \pi^2 \\
&= \pi(1-\pi)e^{-\tau(t-s)} \\
&= \pi(1-\pi)e^{-(\alpha+\beta)(t-s)}
\end{aligned}$$

since the covariance has to be semetric

$$\Rightarrow \; Cov[V(s)V(t)] = \pi(1-\pi)e^{-(\alpha+\beta)|t-s|}$$