

Solution for exercise 6.3.2 in Karlin and Pinsky

One way to model the birth and death process is by stating the transition probabilities and the assumptions made.

A reasonable assumption seems to be that once one of the boundaries is being reached, the beetle will move to the next collar and doesn't leave the system. Therefore we assume reflecting boundaries. Then the transition probabilities become:

$$P(X(t+h) = i | X(t) = j) = \begin{cases} 1 - \frac{1}{m_j} \cdot h + o(h) & i = j \\ \frac{1}{2m_j} \cdot h + o(h) & i = j + 1, j \neq 0, j \neq N \\ \frac{1}{2m_j} \cdot h + o(h) & i = j - 1, j \neq 0, j \neq N \\ \frac{1}{m_j} \cdot h + o(h) & i = j - 1, j = N \\ \frac{1}{m_j} \cdot h + o(h) & i = j + 1, j = 0 \end{cases}$$