IMM - DTU

02407 Stochastic Processes
2023-10-2
DAME,BFN/dame,bfn

Solution for exercise 6.2.1 in Karlin and Pinsky

$$
\begin{aligned}
P(X(T)=0) &= P\left(W_N < T\right) = P\left(\sum_{i=1}^{N} S_i < T\right) \\
&= P\left(S_N < T\right) P\left(\sum_{i=1}^{N} S_i < T \,\middle|\, S_N < T\right) \\
&= P\left(S_N < T\right) P\left(\sum_{i=1}^{N-1} S_i < T - S_N \,\middle|\, S_N < T\right) \\
&= P\left(S_N < T\right) P\left(\sum_{i=1}^{N-1} S_i < T\right) \qquad (1) \\
&= \prod_{i=1}^{N} P\left(S_i < T\right) \\
&= \prod_{i=1}^{N} \frac{\mu_i}{\mu_i + \theta}
\end{aligned}
$$

1: independent and memoryless