IMM - DTU

02407 Stochastic Processes
2011-9-27
DAME/dame

Solution for exercise 6.1.4 in Karlin and Pinsky

a)

$$\begin{aligned}\lambda_0 &= \alpha = 1\\ \lambda_k &= k\theta + \alpha = k \cdot 2 + 1\end{aligned}$$

b)

$$\lambda_0 = 1, \quad \lambda_1 = 3, \quad \lambda_2 = 5$$

$\lambda_i \neq \lambda j$ for all $i \neq j$ implies equation 6.8 holds.

$$\begin{aligned}P_2(1) &= \lambda_0 \cdot \lambda_1 \cdot \left[\beta_{0,2} e^{-\lambda_0} + \beta_{1,2} e^{-\lambda_1 + \beta_{2,2} e^{-\lambda_2}}\right]\\ &= \alpha(\alpha+\theta)\left[\frac{1}{\lambda_1 - \lambda_0} \cdot \frac{1}{\lambda_2 - \lambda_0} e^{-\lambda_0} + \frac{1}{\lambda_0 - \lambda_1} \cdot \frac{1}{\lambda_3 - \lambda_1} e^{-\lambda_1} + \frac{1}{\lambda_0 - \lambda_3} \cdot \frac{1}{\lambda_1 - \lambda_3} e^{-\lambda_3}\right]\\ &= 1 \cdot 3 \left[\frac{1}{2}\frac{1}{4} e^{-1} + \left(-\frac{1}{2}\right)\frac{1}{2} e^{-3} + \left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right) e^{-5}\right]\\ &= \frac{3}{4}\left[\frac{1}{2} e^{-1} - e^{-3} + \frac{1}{2} e^{-5}\right]\\ &\doteq 0.10314\end{aligned}$$