

Solution for exercise 5.4.7 in Karlin and Pinsky

The W_i are iid, so we know from chapter 2, equation 2.30

$$\begin{aligned} E\left[\sum_{i=0}^{X(t)} f(W_i)\right] &= E[X(t)]E[f(W_1)] \\ &= \lambda t E[f(W_1)] \\ &= \lambda \int_0^t f(w)dw \end{aligned}$$