

## Solution for problem 5.1.9 in Karlin and Pin- sky

### 0.1 a)

$$\begin{aligned}E[X(T)|T = t] &= E[X(t)] = \lambda \cdot t = 2t \\ \text{Var}[X(T)|T = t] &= \text{Var}[X(t)] = \lambda \cdot t = 2t \\ E[X^2(T)|T = t] &= \text{Var}[X(T)|T = t] + (E[X(T)|T = t])^2 \\ &= 2t + 4t^2 = 2t(1 + 2t)\end{aligned}$$

### 0.2 b)

$$\begin{aligned}E[X(T)] &= E[E[X(T)|T]] = E[2T] = \int_0^1 2t \cdot 1 \, dt = 1 \\ E[X^2(T)] &= E[E[X^2(T)|T]] = E[2T + 4T^2] = \int_0^1 (2t + 4t^2) \cdot 1 \, dt = \frac{7}{3} \\ \text{Var}[X(T)] &= E[X^2(T)] - (E[X(T)])^2 = \frac{4}{3}\end{aligned}$$