IMM - DTU

02407 Stochastic Processes
2011-9-20
DAME/dame

Solution for exercise 5.1.7 in Karlin and Pinsky

Let $S(t)$ be the event that the system is surviving at time t. Furthermore let $N(t)$ be the number of shocks occuring until time t and $p_k = \alpha^k$ the probabilty that the system survives k shocks. The distribution of the shocks is poisson with rate λ and we know $P(N(t) = k) = \frac{\lambda^k}{k!}e^{-\lambda}$

$$\begin{aligned}
P(S(t)) &= \sum_{k=0}^{\infty} P(S(t)|N(t) = k) \cdot P(N(t) = k) \\
&= \sum_{k=0}^{\infty} p_k \cdot P(N(t) = k) \\
&= \sum_{k=0}^{\infty} \alpha^k \cdot P(N(t) = k) \\
&= \sum_{k=0}^{\infty} \alpha^k \cdot \frac{\lambda^k}{k!} e^{-\lambda} \\
&= e^{-\lambda(1-\alpha)}
\end{aligned}$$