Solution for exercise 5.1.7 in Karlin and Pinsky

Let \( S(t) \) be the event that the system is surviving at time \( t \). Furthermore let \( N(t) \) be the number of shocks occurring until time \( t \) and \( p_k = \alpha^k \) the probability that the system survives \( k \) shocks. The distribution of the shocks is poisson with rate \( \lambda \) and we know \( P(N(t) = k) = \frac{\lambda^k}{k!}e^{-\lambda} \)

\[
P(S(t)) = \sum_{k=0}^{\infty} P(S(t)|N(t) = k) \cdot P(N(t) = k)
\]
\[
= \sum_{k=0}^{\infty} p_k \cdot P(N(t) = k)
\]
\[
= \sum_{k=0}^{\infty} \alpha^k \cdot P(N(t) = k)
\]
\[
= \sum_{k=0}^{\infty} \alpha^k \cdot \frac{\lambda^k}{k!}e^{-\lambda}
\]
\[
= e^{-\lambda(1-\alpha)}
\]