IMM - DTU

02407 Stochastic Processes
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Solution for exercise 4.4.7 in Karlin and Pinsky

First we have to consider which states are possible. It is possible that the individual has the car where he is or not. At the same time it is possible that it rains as well that the sun shines. Let the possible combinations be $\{(c,r),(c,s),(nc,r),(nc,s)\}$ c standing for car, nc for no car and r for rain as well as s for sun. Than the transition matrix has the form

$$P = \begin{array}{c|cccc|} & \text{(c,r)} & \text{(c,s)} & \text{(nc,r)} & \text{(nc,s)} \\ \hline \text{(c,r)} & \text{p} & \text{1-p} & 0 & 0 \\ \text{(c,s)} & 0 & 0 & \text{p} & \text{1-p} \\ \text{(nc,r)} & \text{p} & \text{1-p} & 0 & 0 \\ \text{(nc,s)} & \text{p} & \text{1-p} & 0 & 0 \end{array}$$

With stationary distribution $\begin{pmatrix} \frac{p}{2-p} & \frac{1-p}{2-p} & \frac{p(1-p)}{2-p} & \frac{(1-p)^2}{2-p} \end{pmatrix}$. So the man walks $\pi_3 = \frac{p(1-p)}{2-p}$ fraction of the time in the rain.

Assume now the man has two cars than the state space has to be adapted.

$$P = \begin{array}{c|cccccc|} & \text{(c,c,r)} & \text{(c,c,s)} & \text{(c,nc,r)} & \text{(c,nc,s)} & \text{(nc,nc,r)} & \text{(nc, nc, s)} \\ \hline \text{(c,c,r)} & 0 & 0 & \text{p} & \text{1-p} & 0 & 0 \\ \text{(c,c,s)} & 0 & 0 & 0 & 0 & \text{p} & \text{1-p} \\ \text{(c,nc,r)} & \text{p} & \text{1-p} & 0 & 0 & 0 & 0 \\ \text{(c,nc,s)} & 0 & 0 & \text{p} & \text{1-p} & 0 & 0 \\ \text{(nc,nc,r)} & \text{p} & \text{1-p} & 0 & 0 & 0 & 0 \\ \text{(nc,nc,s)} & \text{p} & \text{1-p} & 0 & 0 & 0 & 0 \end{array}$$

With stationary distribution $\begin{pmatrix} \frac{p}{3-p} & \frac{1-p}{3-p} & \frac{p}{3-p} & \frac{1-p}{3-p} & \frac{p(1-p)}{3-p} & \frac{(1-p)^2}{3-p} \end{pmatrix}$. So the man walks $\pi_5 = \frac{p(1-p)}{3-p}$ fraction of the time in the rain.