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Solution for exercise 4.4.3 in Karlin and Pinsky

The Markov Chain in 4.4.3 is a reversible Markov chain and we can get the following relations

$$\pi_{i-1}p_{i-1} = q_i\pi_i$$

and therefore

$$\pi_{i} = \prod_{k=0}^{i-1} \frac{p_{k}}{q_{k+1}} \pi_{0} \ \forall 1 \leq i \leq N-1$$

$$\pi_{N} = p_{N-1} \cdot \pi_{N-1} = p_{N-1} \prod_{k=0}^{N-1} p_{i_{0}} = \frac{1}{\sum_{i=0}^{N-1} \prod_{k=0}^{i-1} \frac{p_{k}}{q_{k+1}} + p_{N-1} \prod_{k=0}^{N-1} p_{i_{0}}}$$