DTU Compute

02407 Stochastic Processes
sep 17 2018
DAME,JFAN

Solution for problem 4.3.3 in Karlin and Pinsky

Using the transition matrix we can easily see $f_{00}^{(1)} = P(X_1 = 0|X_0 = 0) = P_{00}^{(1)} = 0$. The other cases are found using eq. (4.16)

$$
\begin{aligned}
P_{00}^{(2)} &= \frac{1}{4} = f_{00}^{(0)}P_{00}^{(2)} + f_{00}^{(1)}P_{00}^{(1)} + f_{00}^{(2)}P_{00}^{(0)} = f_{00}^{(2)} \\
P_{00}^{(3)} &= \frac{1}{8} = f_{00}^{(0)}P_{00}^{(3)} + f_{00}^{(1)}P_{00}^{(2)} + f_{00}^{(2)}P_{00}^{(1)} + f_{00}^{(3)}P_{00}^{(0)} = f_{00}^{(3)} \\
P_{00}^{(4)} &= \frac{3}{8} = f_{00}^{(0)}P_{00}^{(4)} + f_{00}^{(1)}P_{00}^{(3)} + f_{00}^{(2)}P_{00}^{(2)} + f_{00}^{(3)}P_{00}^{(1)} + f_{00}^{(4)}P_{00}^{(0)} \\
&= \frac{1}{4}\frac{1}{4} + f_{00}^{(4)} \Leftrightarrow f_{00}^{(4)} = \frac{5}{16} \\
P_{00}^{(5)} &= \frac{7}{32} = f_{00}^{(0)}P_{00}^{(5)} + f_{00}^{(1)}P_{00}^{(4)} + f_{00}^{(2)}P_{00}^{(3)} + f_{00}^{(3)}P_{00}^{(2)} + f_{00}^{(4)}P_{00}^{(1)} + f_{00}^{(5)}P_{00}^{(0)} \\
&= \frac{1}{4}\frac{1}{8} + \frac{1}{8}\frac{1}{4} + f_{00}^{(5)} \Leftrightarrow f_{00}^{(5)} = \frac{5}{32}
\end{aligned}
$$