IMM - DTU

02407 Stochastic Processes
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DAME/dame

Solution for exercise 4.3.2 in Karlin and Pinsky

we have to show that a finite-state aperiodic irreducible Markov chain is regular and recurrent. If we consider an finite state Markov chain then at least one state is recurrent, since there are only finite possible states at least one of them has to be visited infinitely often.

Furthermore we know that we are looking at an irreducible Markov chain, recurrent is a class property, therefore we know the Markov chain is recurrent. Since the Markov chain is aperiodic we know that there an integer N depending on i such that for all $n \geq N$ we get $P_{ii}^{n} > 0$. This means $\exists M \in \mathbb{N}$ with $P_{ii}^{m} > 0$ for all i and $m \geq M$. Since the Markov chain is irreducible and recurrent we can follow,there exist a $\tilde{M}$ with with $P_{ij}^{\tilde{m}} > 0$ for all i, j, and $\tilde{m} > \tilde{M}$