Solution for exercise 4.1.13 in Karlin and Pinsky

The limiting distribution of the Markov Chain is $\pi = \left(\frac{11}{24} \quad \frac{7}{24} \quad \frac{6}{24} \right)$ Furthermore

$$
\begin{aligned}
\lim_{n \to \infty} P(X_{n-1} = 2 | X_n = 1) &= \lim_{n \to \infty} \frac{P(X_n = 1 | X_{n-1} = 2) \cdot P(X_{n-1} = 2)}{P(X_n = 1)} \\
&= P_{21} \cdot \lim_{n \to \infty} \frac{P(X_{n-1} = 2)}{P(X_n = 1)} \\
&= \frac{1}{5} \cdot \frac{\pi_2}{\pi_1} \\
&= \frac{6}{35}
\end{aligned}
$$