02407 Stochastic Processes 2011-9-14 DAME/dame

Solution for exercise 4.1.13 in Karlin and Pinsky

The limiting distribution of the Markov Chain is $\pi = \begin{pmatrix} \frac{11}{24} & \frac{7}{24} & \frac{6}{24} \end{pmatrix}$ Furthermore

$$\lim_{n \to \infty} P(X_{n-1} = 2 | X_n = 1) = \lim_{n \to \infty} \frac{P(X_n = 1 | X_{n-1} = 2) \cdot P(X_{n-1} = 2)}{P(X_n = 1)}$$

$$= P_{21} \cdot \lim_{n \to \infty} \frac{P(X_{n-1} = 2)}{P(X_n = 1)}$$

$$= \frac{1}{5} \cdot \frac{\pi_2}{\pi_1}$$

$$= \frac{6}{35}$$