Solution for exercise 4.1.12 in
Karlin and Pinsky

0.1  a)

It has to be shown that $P^n = Q^n + \Pi$ with $Q = P - \Pi$.
Since $\Pi$ has the stationary distribution as row vectors and $P$ is a stochastic
matrix the row sums are equal to one.
Therefore $\Pi \cdot P = \Pi$, $P \cdot \Pi = \Pi$ and $\Pi \cdot \Pi = \Pi$
Furthermore we use that $P^n = Q^n + \Pi \iff P^n - \Pi = Q^n$

\[
Q^2 = (P - \Pi)^2 \\
= P^2 - P\Pi - \Pi P + \Pi^2 \\
= P^2 - \Pi - \Pi + \Pi \\
= P^2 - \Pi
\]

Assuming $Q^{n-1} = P^{n-1} - \Pi$ the equation has only to be proofen for $Q^n$

\[
Q^n = Q^{n-1}Q \\
= (P^{n-1} - \Pi)(P - \Pi) \\
= P^n - P^{n-1}\Pi - \Pi P + \Pi^2 \\
= P^n - \Pi - \Pi + \Pi \\
= P^n - \Pi \\
\iff P^n = Q^n + \Pi
\]
0.2  b)

With \( \pi = \left( \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \right) \) we get:

\[
Q = \begin{pmatrix}
\frac{1}{4} & 0 & -\frac{1}{4} \\
0 & 0 & 0 \\
-\frac{1}{4} & 0 & \frac{1}{4}
\end{pmatrix}
\]

\[
Q^n = \begin{pmatrix}
\frac{1}{4}2^{n-1} & 0 & -\frac{1}{4}2^{n-1} \\
0 & 0 & 0 \\
-\frac{1}{4}2^{n-1} & 0 & \frac{1}{4}2^{n-1}
\end{pmatrix}
\]

\[
P^n = \begin{pmatrix}
\frac{1}{4}2^{n-1} + \frac{1}{4} & \frac{1}{2} & -\frac{1}{4}2^{n-1} + \frac{1}{4} \\
\frac{1}{4}2^{n-1} + \frac{1}{4} & \frac{1}{2} & \frac{1}{4}2^{n-1} + \frac{1}{4} \\
-\frac{1}{4}2^{n-1} + \frac{1}{4} & \frac{1}{2} & \frac{1}{4}2^{n-1} + \frac{1}{4}
\end{pmatrix}
\]