3.9.7

$$\phi(s) = \frac{1}{2}\sum_{l=0}^{\infty}(\frac{1}{2})^l s^l - \frac{1}{4} + \frac{1}{4}s$$

If we calculate the expectation directly we get:

$$\begin{aligned}
E[X] &= \sum_{k=0}^{\infty} k(\frac{1}{2})^k \frac{1}{2} + \frac{1}{4} \\
&= \sum_{k=1}^{\infty} k(\frac{1}{2})^k \frac{1}{2} + \frac{1}{4} \\
&= \sum_{k=0}^{\infty} (k+1)(\frac{1}{2})^{k+1} \frac{1}{2} + \frac{1}{4} \\
&= \sum_{k=0}^{\infty} k(\frac{1}{2})^{k+1} \frac{1}{2} + \frac{1}{4} + \sum_{k=0}^{\infty} (\frac{1}{2})^{k+1} \frac{1}{2} \\
&= \frac{1}{2}\sum_{k=0}^{\infty} k(\frac{1}{2})^k \frac{1}{2} + \frac{3}{4} \\
\rightarrow E[X] &= \frac{5}{4}
\end{aligned}$$

The two infinte sums are the expectation of an geometric distributed random variable as well the sum of a geometric series. Differentiating $\phi(s)$ and evaluating it in $s = 1$ we get:

$$E[X] = \frac{1}{2}\sum_{l=0}^{\infty} l(\frac{1}{2})^l + \frac{1}{4} = \frac{1}{2}\sum_{l=0}^{\infty} l(\frac{1}{2})^l \frac{1}{2} + \frac{1}{4}$$

similar calculations as before delivere the same result.