IMM - DTU

02407 Stochastic Processes
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DAME,BFN/dame,bfn

Solution for exercise 3.7.1 in Karlin and Pinsky

The transition probability matrix $\boldsymbol{P}$ partitions as

$$\boldsymbol{P} = \begin{pmatrix} 1 & \vec{0} \\ \vec{q} & \boldsymbol{Q} \end{pmatrix}$$

where the elements q_i of $\vec{q}$ are $q_i = 1/i$ and the elements Q_{ij} of $\boldsymbol{Q}$ are $Q_{ij} = 1/i$ for $j < i$ and 0 otherwise. To calculate $\boldsymbol{W} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$ we first state $\boldsymbol{W}^{-1} = \boldsymbol{I} - \boldsymbol{Q} = \{w_{ij}^{-1}\}$ $(1 \leq j, i)$

$$w_{ij}^{-1} = \begin{cases} 0 & j > i \\ 1 & j = i \\ -\frac{1}{i} & j < i \end{cases}$$

Now we have to calculate $(\boldsymbol{W}^{-1})^{-1} = \boldsymbol{W}$ with elements w_{ij}.

$$w_{ij} = \begin{cases} 0 & j > i \\ 1 & j = i \\ \frac{1}{j+1} & j < i \end{cases}$$

We derive the inverse recursively due to the simple structure of $\boldsymbol{W}$.