# 3.6.2

Problem 3.6.2 is referring to a random walk with transitition matrix like given on page 128 in Pinsky/Karlin.
Obviously $v_0 = E[T|X_0 = 0] = 0$ and $v_N = E[T|X_0 = N] = 0$. Making a first step analysis we get:

$$\begin{aligned} v_k &= q_k(1+v_{k-1}) + r_k(1+v_k) + p_k(1+v_{k+1}) \\ &= 1(q_k + r_k + p_k) + q_k v_{k-1} + r_k v_k + p_k v_{k+1} \end{aligned}$$

In order to derive the other equations we would have to consider

$$\begin{aligned} v_k &= q_k(1+v_{k-1}) + r_k(1+v_k) + p_k(1+v_{k+1}) \\ (q_k + r_k + p_k)v_k &= q_k(1+v_{k-1}) + r_k(1+v_k) + p_k(1+v_{k+1}) \\ \rightarrow \ v_{k+1} - v_k &= \frac{q_k}{p_k}(v_k - v_{k-1}) - \frac{1}{p_k} \end{aligned}$$

Define $X_k := v_{k+1} - v_k$. therefore the formula can be rewritten as:

$$\begin{aligned} X_k &= \frac{q_k}{p_k} X_{k-1} - \frac{1}{p_k} \\ &= \prod_{i=1}^{k} \frac{q_k}{p_k} X_0 - \sum_{l=1}^{k-1} \prod_{j=0}^{l-1} \frac{q_{k-j}}{p_{k-j}} \end{aligned}$$

The trckis now to write $v_{k+1} = \sum_{l=1}^{k} X_k$ and to fullfill some algebraic calculations. We expect the expected time till absorbtion to be higher than in the case where $r_k = 0$