Solution for problem 3.4.7 in Pinsky and Karlin

\[
h_i = E[\sum_{n=0}^{\infty} \beta^n \cdot c(X_n)|X_0 = i]
\]

\[
= E[\beta^0 c(X_0)|X_0 = i] + E[\sum_{n=1}^{\infty} \beta^n \cdot c(X_n)|X_0 = i]
\]

\[
= c(i) + \sum_j E[\sum_{n=1}^{\infty} \beta^n \cdot c(X_n)|X_0 = i, X_1 = j]p_{ij}
\]

\[
= c(i) + \sum_j E[\sum_{n=0}^{\infty} \beta^{n+1} \cdot c(X_{n+1})|X_1 = j]p_{ij}
\]

\[
= c(i) + \sum_j E[\sum_{n=0}^{\infty} \beta^{n+1} \cdot c(X_n)|X_0 = j]p_{ij}
\]

\[
= c(i) + \beta \sum_j h_j p_{ij}
\]

Third equation holds due to the law of total expectation.