DTU Compute

02407 Stochastic Processes
3 Sep 2018
DAME/BFN/JFAN

Solution for problem 3.4.7 in Pinsky and Karlin

$$
\begin{aligned}
h_i &= E[\sum_{n=0}^{\infty} \beta^n \cdot c(X_n) | X_0 = i] \\
&= E[\beta^0 c(X_0) | X_0 = i] + E[\sum_{n=1}^{\infty} \beta^n \cdot c(X_n) | X_0 = i] \\
&= c(i) + \sum_j E[\sum_{n=1}^{\infty} \beta^n \cdot c(X_n) | X_0 = i \ X_1 = j] p_{ij} \\
&= c(i) + \sum_j E[\sum_{n=0}^{\infty} \beta^{n+1} \cdot c(X_{n+1}) | \ X_1 = j] p_{ij} \\
&= c(i) + \sum_j E[\sum_{n=0}^{\infty} \beta^{n+1} \cdot c(X_n) | \ X_0 = j] p_{ij} \\
&= c(i) + \beta \sum_j h_j p_{ij}
\end{aligned}
$$

Third equation holds due to the law of total expectation.