DTU Compute

02407 Stochastic Processes
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Solution for problem 3.4.4 in Pinsky and Karlin

Denote V_2 as the event that state 2 is visited before absorption. The probability in question is $\mathbb{P}(V_2^c)$. To calculate this probability we will consider the Markov chain

$$\boldsymbol{P}' = \left(\begin{array}{c|cccc} & 0' & 1' & 2' & 3' \\ \hline 0' & 0.2 & 0.2 & 0.1 & 0.5 \\ 1' & 0.2 & 0.3 & 0.2 & 0.3 \\ 2' & 0 & 0 & 1 & 0 \\ 3' & 0 & 0 & 0 & 1 \end{array} \right),$$

where state 0 in the original chain corresponds to state 2' in the new chain, state 1 in the original chain corresponds to state 0' in the new chain and state 3 in the original chain corresponds to state 1' in the new chain. State 2 from the original chain has been replaced with the absorbing state 3'. The event that the new chain gets absorbed in state 3' is the same as the probability that the original chain visits state 2 at least once. We can thus calculate the $\mathbb{P}(V_2^c)$ by calculating the probability that the chain with probability transition matrix $\boldsymbol{P}'$ gets absorbed in state 2' rather than 3'. We do this using the standard terminology of the text book

$$\begin{aligned} u_{0'} &= 0.1 + 0.2u_{0'} + 0.2u_{1'} \\ u_{1'} &= 0.2 + 0.2u_{0'} + 0.3u_{1'} \end{aligned}$$

leading to

$$u_{0'} = \mathbb{P}(V_2^c) = \frac{11}{52}$$