IMM - DTU

02407 Stochastic Processes
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DAME/BFN

Solution for exercise 3.4.12 in Pinsky and Karlin

Let T be the time of absorption and define $z_i := Pr(X_{T-1} = 1 | X_0 = i)$. This can be used for a first step analysis:

$$\begin{aligned} z_0 &= 0.3z_0 + 0.2z_1 \\ z_1 &= 0.4 + 0.5z_0 + 0.1z_1 \\ \rightarrow\ z_0 &= \frac{8}{53} \end{aligned}$$

Alternatively we could introduce a second absorbing state, such that the modified chain would be

$$\boldsymbol{P} = \left(\begin{array}{c|cccc} & 0 & 1 & 2 & 2' \\ \hline 0 & 0.3 & 0.2 & 0.5 & 0 \\ 1 & 0.5 & 0.1 & 0 & 0.4 \\ 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

Defining $u_i = Pr(X_{T-1} = 2' | X_0 = i)$ we find

$$\begin{aligned} u_0 &= 0.3u_0 + 0.2u_1 \\ u_1 &= 0.4 + 0.5u_0 + 0.1u_1 \end{aligned}$$

leading to $u_0 = \frac{8}{53}$ which is seen to be similar to z_0. A short reflection gives that z_0 and u_0 is two different ways of evaluating the probability of the event that absorption occurs from state 1. The former is slightly faster and elegant, while the second method is obtained by directly using the formulation of Section 3.4.