

Solution for problem 3.2.5 in Pinsky and Karlin

$$P^3 = \begin{pmatrix} 0,4570 & 0,2300 & 0,3130 \\ 0,3450 & 0,2270 & 0,4280 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} P(X_3 = 0 | X_0 = 0, T > 3) &= P(X_3 = 0 | X_0 = 0, \{X_3 = 0\} \cup \{X_3 = 1\}) \\ &= \frac{P(X_3 = 0 \cap (\{X_3 = 0\} \cup \{X_3 = 1\}) | X_0 = 0)}{P(\{X_3 = 0\} \cup \{X_3 = 1\} | X_0 = 0)} \\ &= \frac{P(X_3 = 0 | X_0 = 0)}{P(X_3 = 0 | X_0 = 0) + P(X_3 = 1 | X_0 = 0)} \\ &= \frac{0,457}{0,457 + 0,230} = 0,6652 \end{aligned}$$